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155. Proposed by R. D. BOHANNON, Ph. D., Professor of Mathematics, Ohio State University, Columbus, O.

If  $\frac{x}{a+\alpha} + \frac{y}{b+\alpha} + \frac{z}{c+\alpha} = \frac{x}{a+\beta} + \frac{y}{b+\beta} + \frac{z}{c+\beta} = \frac{x}{a+\gamma} + \frac{y}{b+\gamma} + \frac{z}{c+\gamma} = 1$ , show,

$$\frac{x}{(a+\beta)^2} + \frac{y}{(b+\beta)^2} + \frac{z}{(c+\beta)^2} = \frac{(\gamma-\beta)(\alpha-\beta)}{(a+\beta)(b+\beta)(c+\beta)}.$$

Solution by G. E. M. ZERR, A.M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Consider the equation,

$$\frac{x}{a+\varphi} + \frac{y}{b+\varphi} + \frac{z}{c+\varphi} = 1 - \frac{(\varphi-\alpha)(\varphi-\beta)(\varphi-\gamma)}{(a+\varphi)(b+\varphi)(c+\varphi)}.$$

Multiply through by  $a+\varphi$  and then put  $a+\varphi=0$ .

$$\therefore x = \frac{(a+\alpha)(a+\beta)(a+\gamma)}{(\alpha-\beta)(\alpha-\gamma)}, \quad y = \frac{(b+\alpha)(b+\beta)(b+\gamma)}{(\beta-\gamma)(\beta-\alpha)}, \quad z = \frac{(c+\alpha)(c+\beta)(c+\gamma)}{(\gamma-\alpha)(\gamma-\beta)}$$

$$\begin{aligned} \therefore \frac{x}{(a+\beta)^2} + \frac{y}{(b+\beta)^2} + \frac{z}{(c+\beta)^2} &= \frac{(a+\alpha)(a+\gamma)}{(a+\beta)(\alpha-\beta)(\alpha-\gamma)} \\ &+ \frac{(b+\alpha)(b+\gamma)}{(b+\beta)(\beta-\gamma)(\beta-\alpha)} + \frac{(c+\alpha)(c+\gamma)}{(c+\beta)(\gamma-\alpha)(\gamma-\beta)} = \frac{(\gamma-\beta)(\alpha-\beta)}{(a+\beta)(b+\beta)(c+\beta)}. \end{aligned}$$

$$\text{Also } \frac{x}{(a+\alpha)^2} + \frac{y}{(b+\alpha)^2} + \frac{z}{(c+\alpha)^2} = \frac{(\gamma-\alpha)(\beta-\alpha)}{(a+\alpha)(b+\alpha)(c+\alpha)}.$$

$$\frac{x}{(a+\gamma)^2} + \frac{y}{(b+\gamma)^2} + \frac{z}{(c+\gamma)^2} = \frac{(\alpha-\gamma)(\beta-\gamma)}{(a+\gamma)(b+\gamma)(c+\gamma)}.$$

Also solved by LON C. WALKER.

159. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

If  $x-1=3m$ ,  $x^2-1=4n$ ,  $x^3-1=5p$ , where  $m, n, p$  are integers, find a general expression for  $x$ .

I. Solution by L. E. DICKSON, Ph. D., The University of Chicago, Chicago, Ill.

The required expression for  $x$  is of the form  $3m+1$ , where  $m$  is subject to the conditions that  $(3m+1)^2-1$  shall be divisible by 4, and that  $(3m+1)^3-1$  shall be divisible by 5. The first condition is satisfied if, and only if,  $m$  is even. The second condition requires that either  $m$  or else  $3m^2+3m+1$  shall be divisible by 5. To prove that the last alternative is impossible, we note that

$$2(3m^2+3m+1) \equiv (m+3)^2 - 2 \pmod{5},$$

and hence is not  $\equiv 0 \pmod{5}$ , 2 being a non-quadratic residue of 5. The necessary and sufficient conditions are, therefore, that  $m$  shall be divisible by 10, so that  $x$  shall have the form  $x=30t+1$ .

II. Solution by MARCUS BAKER, U. S. Geological Survey, Washington, D. C.

From the three equations we readily obtain

$$x-1=3m \qquad \qquad \qquad =3m,$$

$$x^2-1=3m(2+3m) \qquad \qquad \qquad =4n,$$

$$x^3-1=9m[1+3m(1+m)]=5p;$$

$$\text{whence } n=\frac{3}{4}m(2+3m) \text{ and } p=\frac{3}{5}m[1+3m(1+m)],$$

where  $m$  must be so taken as to make  $n$  and  $p$  integers. To make  $n$  integral  $m$  must be an *even* number; to make  $p$  integral  $m$  must be a multiple of 5; hence  $m=10k$  where  $k$  may be any positive integer. Hence

$$m=10k,$$

$$n=15k(1+15k),$$

$$p=18k[1+30k(1+10k)].$$

$$\text{Also } x-1=30k,$$

$$x^2-1=60k(1+15k),$$

$$x^3-1=90k[1+30k(1+10k)];$$

$$\text{and } x=1, 31, 61, 91, \text{ etc.}$$

Solved similarly by G. B. M. ZERR, LON C. WALKER, and HON. J. H. DRUMMOND.

160. Proposed by J. SCHEFFER, A. M., Hagerstown, Mo.

Represent the square root of  $10+2\sqrt{6}+2\sqrt{10}+2\sqrt{15}$  as the sum of three square roots.

Solution by G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.; LON C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University; and the late HON. JOSIAH H. DRUMMOND.

$$\text{Let } \sqrt{10+2\sqrt{6}+2\sqrt{10}+2\sqrt{15}}=\sqrt{x}+\sqrt{y}+\sqrt{z}.$$

$$\therefore 10+2\sqrt{6}+2\sqrt{10}+2\sqrt{15}=x+y+z+2\sqrt{xy}+2\sqrt{xz}+2\sqrt{yz}.$$

$$\therefore 2\sqrt{xy}=2\sqrt{6}, \therefore \sqrt{xy}=\sqrt{6}. \text{ Similarly, } \sqrt{xz}=\sqrt{10}, \sqrt{yz}=\sqrt{15}.$$

$$\therefore \sqrt{(xyz)}=\sqrt{30}. \therefore \sqrt{x}=\sqrt{2}, \sqrt{y}=\sqrt{3}, \sqrt{z}=\sqrt{5}.$$

$\therefore \sqrt{10+2\sqrt{6}+2\sqrt{10}+2\sqrt{15}}=\sqrt{2}+\sqrt{3}+\sqrt{5}=\text{the sum of the three square roots.}$